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COMPUTER SIMULATION OF BALANCE HANDLING

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Abstract

Several decades ago, when working in the field of magnetism, we had to use a balance the sensitivity of which was limited only by Brownian motion. This balance was a very slow one and to calculate the moment of force measured by it we used its equation of motion, $T=J\ddot{\alpha}+k\dot{\alpha}+C\alpha$, where we measured the values of all the quantities present on the right-hand side of this equation. At the 21st Conference on Vacuum Microbalance Techniques in Dijon, we suggested that, with the help of a computer, this procedure could also be made applicable to the handling of fast balances. The present paper contributes to this topic by presenting a computer simulation of such a fast balance.

Keywords: balance, computer, mass determination, simulation

Some thirty years ago, we worked with a balance which had such a high sensitivity that its accuracy was limited only by the Brownian motion [1, 2]. This balance was very slow: the oscillation frequency ω was very small and the relaxation time τ was very large. To allow measuring times of a few seconds, we used Eq. (1) of Table 1. We measured the deflection angle α as a function of time and calculated $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$. The values of the moment of rotational inertia J, the damping constant k and the torsion constant C were known and so we could deduce the moment of force $T(t)$ as a function of time from Eq. (1), where $T(t)$ is the sum of the torque to be measured and the compensating torque.

In another paper in this volume [3], we discussed the possibility of applying this measurement procedure to beam microbalances in order to reduce the measuring time in that case to only a small fraction of the oscillating time. This procedure, however, involves uncertainties, as we discussed in that paper.

For our very slow balance mentioned above, we reduced such uncertainties by application of an iteration method. This iteration involved the reduction of the balance deviation from the equilibrium position. To minimize the time interval

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necessary for this reduction, for the compensating torque we used two opposite pulses with a short interval between them. It may be expected that such an iteration method should also be applicable for faster beam balances.

We studied this by means of the computer simulation depicted in Table 1. The data used there are taken from a Gast vacuum microbalance with 2.5 g maximum capacity, which is used at present by one of the authors for measuring gas adsorption [4]. In Table 1, the equations and parameters on the left-hand side describe the working of the balance, and those on the right-hand side its automation. The motion of the balance is expressed by Eq. (2). Here, the amplitude $\hat{\alpha}$ and phase angle φ can be chosen arbitrarily, and the moment of torque T is assumed to be independent of time. The relation between the parameters of Eq. (2) and those of Eq. (1) are expressed by Eqs (3) and (4), in which ω denotes the angular frequency and τ the relaxation time. Equations (5), (6), (7) and (9) give the values of the different parameters. It is the aim of the procedure to find the value of T with the calculation on the right-hand side of the table. We choose the times for measuring the value of α by means of Eq. (8). We presume the measurements of α to be free from error.

The values of α at four times are given in Eq. (16). These values from the balance are the input of the automation facility on the right-hand side of Table 1. The automation facility uses Eqs (14) and (15) to give approximations of the values of $\dot{\alpha}(t)$ and $\ddot{\alpha}(t)$. Inserting Eqs (16) and (17) into Eq. (1) results in an approximation $T(t_2)$ of the moment of force to be determined: Eq. (18).

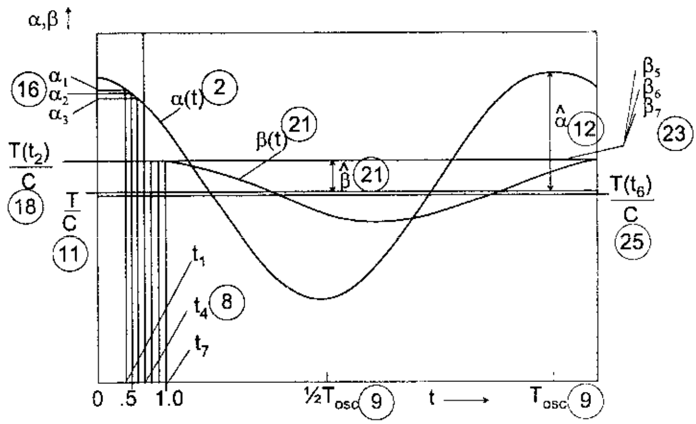


Fig. 1 Weighing procedure. For clearer demonstration, different scales are used on the vertical axis. The encircled numbers refer to the respective equations in Table 1

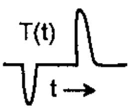


Fig. 2 The two pulses of the compensating torque. See line 19 in Table 1

Table 1 Arithmetic instruction 20-6-97. On the left-hand side, the working of the balance is presented and on the right-hand side the automation.

(1) $T(t) = J\,\ddot{\alpha}(t) + k\,\dot{\alpha}(t) + C\,\alpha(t)$

(2) $\alpha(t) = \frac{T}{C} + \hat{\alpha}\exp\left(-\frac{t}{\tau}\right)\cos(\omega t + \varphi)$

(3,4) $\tau = \frac{2J}{k}$ $\quad \omega = \sqrt{\frac{C}{J} - \frac{k^2}{4J^2}}$

(5,6) $\tau = 200$ s $\quad$ $\boxed{J = 2\cdot 10^{-4}\ \text{kg m}^2,\quad k = 2\cdot 10^{-6}\ \text{kg m}^2\ \text{s}^{-1},\quad C = 2\cdot 10^{-4}\ \text{N m}}$ (6)

(7,8) $\omega = 0.9999875\ \text{s}^{-1}$ $\quad$ $\boxed{t_1 = 0.3\ \text{s},\quad t_n = 0.2 + 0.1\cdot n}$ (8)

(9) $t_{osc} = 6.283264$ s

(10,11) $\varphi = 0$ $\quad$ $\boxed{T = 2\cdot 10^{-5}\ \text{N m}}$

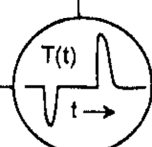
(12) $\hat{\alpha} = 5\cdot 10^{-2}$

(13) $\alpha_n \equiv \alpha(t_n)$

$\dot{\alpha}_n \equiv \frac{\alpha_{n+1} - \alpha_{n-1}}{0.2}$ (14)

$\ddot{\alpha}_n \equiv \frac{\alpha_{n+1} + \alpha_{n-1} - 2\alpha_n}{0.1^2}$ (15)

(16) $\left.\begin{array}{l}\alpha_1 = 0.147695283\\ \alpha_2 = 0.145961132\\ \alpha_3 = 0.143769716\\ \alpha_4 = 0.141143377\end{array}\right\} \rightarrow$ $\quad\rightarrow\quad$ $\left\{\begin{array}{l}\dot{\alpha}_2 = -0.01962783^5\\ \ddot{\alpha}_2 = -0.045726500\end{array}\right.$ (17)



$T(t_2) = 0.000020007671$ (18)

(19)

(20) $\boxed{T(t_2) - T = 7.671\cdot 10^{-9}\ \text{N m}}$ (20)

(21) $\beta(t) = \frac{T}{C} + \frac{T(t_2) - T}{C}\exp\left(-\frac{t - t_4}{\tau}\right)\cos\omega\,(t - t_4)$

(22) $\beta(t) = 10^{-1} + 3.836\cdot 10^{-5}\exp\left(-\frac{t - 0.6}{200}\right)\cdot$
$\cdot\cos\left[0.9999875\,(t - 0.6)\right]$

(23) $\left.\begin{array}{l}\beta_5 = 0.100038149\\ \beta_6 = 0.100037558\\ \beta_7 = 0.100036592\end{array}\right\} \rightarrow$ $\quad\rightarrow\quad$ $\left\{\begin{array}{l}\dot{\beta}_6 = -0.000007785\\ \ddot{\beta}_6 = -0.0000375\end{array}\right.$ (24)

$T(t_6) = 0.00001999985$ (25)

(26) $\boxed{T(t_6) - T = -1.5\cdot 10^{-10}\ \text{N m}}$ (26)

The reason why Eq. (18) deviates from Eq. (10), which represents the real value of T, is that we used the approximate equations (14) and (15). Measurement inaccuracies are ignored in this treatment.

The next step in the iteration procedure is that at time t_4 we apply two short pulses to the compensating torque, and so to $T(t)$ (Fig. 2). These pulses have different signs. The value of $\int T(\mathrm{d}t)$ and the distances in time between the two pulses are chosen such that, after the two pulses, the angle α equals the approximated equilibrium angle $T(t_2)/C$ and that $\dot{\alpha}(t_4)$ equals zero. The difference between t_2 and t_4 causes an extra error besides the error caused by the approximations (14) and (15). Instead of $\alpha(t)$, we shall use the symbol $\beta(t)$ for the deflection after the two pulses, so for $t>t_4$ the balance will satisfy Eqs (21) and (22).

We repeat the measuring procedure at times t_5, t_6 and t_7 and, through calculation of the values of the derivatives $\dot{\beta}$ and $\ddot{\beta}$ of Eq. (24), we get the second estimate of the moment of force to be determined by Eqs (25), (26). We see that subsequent estimates $T(t_2)$ and $T(t_6)$ indeed converge towards the value T of Eq. (11). The time necessary for the complete measurement with two estimates involved can be taken to be $t_7-t_1=0.6$ s according to Eq. (8); this is an order of magnitude smaller than the measurement time with a classical balance governed by Eq. (9).

The experience of the authors is limited to balances which satisfy Eq. (1). For types of balances not satisfying Eq. (1), the iteration method might be questionable.

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